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The heat flux measurement method based on isotherm registration

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Abstract—The heat flux $q_w(t)$ evaluated by using data on isotherms in the coordinate–time plane is considered in this work. The possibility of optical heat flux sensor creation is discussed. The isotherms are recorded by using temperature indicators. The mathematical statement of this problem is formulated as an optimization problem. It is solved with gradient methods and in each iteration the heat equation is solved by the finite-difference method. The results of the numerical experiments are presented. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

The temperature determination method, based on special substances which change their color (or some other property) under heating (temperature indicators) is used in many practical problems [1]. Usually they are located on the bodies' surfaces and retain the maximal temperature values on these surfaces [1]. Sometimes the constant heat flux density is determined if the color change time is registered [1, 2]. The temperature-indicating paints are usually not applied for the measurement of the unsteady-state heat flux because of poor accuracy of $T_w(t)$ determination (caused particularly by the large interval between color change temperatures of different paints).

The heat flux $q_w(t)$ evaluation opportunities provided by using the data on color change zone location depending on time are considered in this work. In particular, we shall consider the scope for optic heat flux sensor creation. Such a sensor can be used in the situation when the use of standard means, based on electric measurements, are not desirable or not possible e.g. for reasons of safety conditions or large electromagnetic influence.

STATEMENT OF THE PROBLEM

Let us consider the strips of temperature-indicating paints embedded along the depth of the material (along the heat flux direction). We use the reversible indicators which recover their color (or other properties) under cooling. It can be liquid crystals or luminescent temperature indicators [1]. One variant of such transparent heat flux sensors is depicted in Fig. 1. It is essential that the sensor material should be transparent in the visual wavelength range and should not be transparent in the infrared (heat) range. It is valid for glass, for example. It allows us not to take into account the radiative heat transfer within the

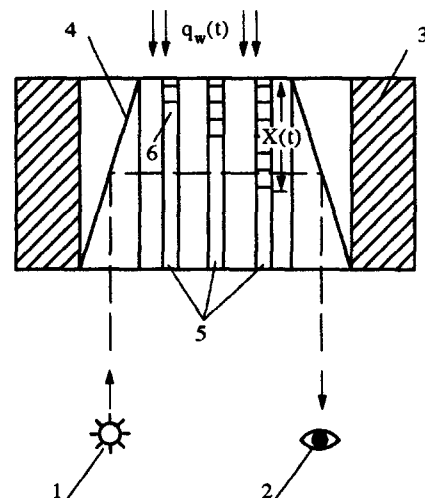


Fig. 1. The heat flux sensor schematic sketch, the temperature indicators and color change zone locations: 1—light source; 2—the signal receptor; 3—heat insulation; 4—mirror; 5—temperature indicators; 6—color change zones.

sensor and, therefore, simplifies the heat transfer model used herein. There should be some special optical system for observing the color change process and a system of storing and analyzing the images. During the measurement process the color change zones of temperature indicators $X_i(t)$ are recorded depending on the time. The $X_i(t)$ curves are the isotherms in two-dimensional space, composed of the spatial and time coordinates (X and t , Fig. 2). It is valid if the temperature field is one-dimensional in the space. This is possible when adequate heat insulation of the sensor lateral sides is achieved.

We shall consider the simplest variant of such sensors, assigned for moderate heat flux values. We use liquid crystal temperature indicators, restricting the sensor admissible temperature range (not higher than 250°C). The low temperature level provides

NOMENCLATURE

t	time [s]
tk	process duration
X	coordinate [m]
$T(t, x)$	temperature [K]
$q_w(t)$	heat flux density [kW m^{-2}]
$C(T)$	heat capacity [$\text{kJ m}^{-3} \text{kg}^{-1} \text{K}^{-1}$]
N	the number of temperature indicators
L	thickness of sensor
T_0	initial temperature.

Greek symbols

ρ	density
$\lambda(T)$	thermal conductivity [$\text{kW m}^{-1} \text{s}^{-1}$]
ε	discrepancy.

Subscripts

comp	computed
exp	experimental
w	wall
i	given indicator number.

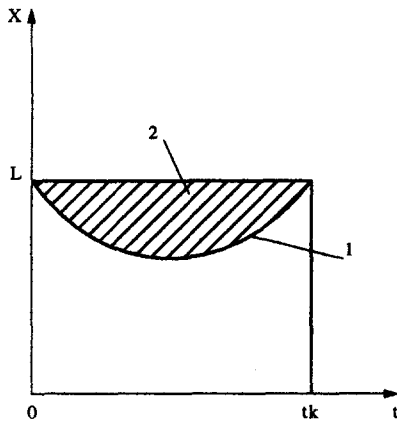


Fig. 2. The color change zones coordinated with time dependence for the one isotherm case (schematic sketch). 1—iso-therm; 2—zone of Cauchy problem solution.

additional reasons for neglecting radiative heat transfer in the transparent sensor structure. The temperature field is considered to be one-dimensional and the effect of thermophysics properties of temperature indicators is neglected too. The measurement range can be significantly enhanced to the higher temperatures by substituting temperature indicators with fusing inserts (the melting zones should be recorded). These inserts should be made from materials with high and precisely known fusing temperatures and should be thin enough. In these cases we use the more complicated model, taking into account the radiative heat transfer in the transparent sensor structure. The consideration of problems arising in this event is beyond the scope of this paper.

THE GOVERNING EQUATIONS

Let us consider the problem of heat flux density $q_w(t)$ evaluation using color change zone coordinates $X_i(t)$ depending on the time (using isotherms $T(t, x) = T_i$).

We write the heat conduction equation with the corresponding initial and boundary conditions:

$$C(T) \cdot \rho \frac{\partial T(t, X)}{\partial t} = \lambda(T) \cdot \frac{\partial^2 T(t, x)}{\partial^2 x} \quad (1)$$

$$\lambda \frac{\partial T}{\partial X} \Big|_{X=L} = q_w(t); \quad \frac{\partial T}{\partial X} \Big|_{X=0} = 0$$

$$X \in (0, L); \quad t \in (0, tk); \quad T(0, X) = T_0(x). \quad (2)$$

We know the set of isotherms corresponding to the temperature indicators' color change temperatures from the experimental data:

$$X|_{T(t)=T_i} = X_i(t), \quad i = 1, \dots, N. \quad (3)$$

We need to find the heat flux $q_w(t)$ in order to reproduce in the computation the experimentally measured values $X_i(t)$.

The given problem is an inverse heat transfer problem [3] and it is, therefore, ill-posed. In the case of one isotherm the solution region is composed of the boundary value problem region (from the heat-insulated surface to the isotherm) and Cauchy problem region (from the heated surface to the isotherm). The configuration of the isotherm and the regions discussed is schematically sketched in Fig. 2. It is the Cauchy problem solution area where our problem is ill-posed.

In the event of two- or three-dimensional temperature fields, the problem statement becomes more complicated because the curves $X_i(t)$ are no longer isotherms and the problem arising is beyond the scope of this paper.

NUMERICAL EXPERIMENTS

In this paper our problem is formulated as an optimization one. The finite difference approximation of the discrepancy gradient is used. The heat flux density vector \mathbf{q}_j ($j = 1 \dots NPT$) parameterizing some function ($\mathbf{q}_j = q(t_j)$) was sought as the vector, minimizing the discrepancy

$$\varepsilon = \sum \int (X_{\text{exp}}(t) - X_{\text{comp}}(t))^2 dt$$

of the computed and experimental values of the color

change zones coordinates $X_i(t)$. The discrepancy gradient was computed by means of q_i small variation and consequent computation of value ε . The steepest descent method and the method of conjugated gradients were used as the optimization algorithms. The multi-fold direct problem (equations (1) and (2)) solution is the reason for high computer time consumption for solution of the problem. The model problem computations for a specimen having the characteristics $\lambda = 0.00074 \text{ kW (m s)}^{-1}$; $C \cdot \rho = 2500 \text{ kJ m}^{-3}$; thickness 0.01 m were made in order to confirm the validity of the above-discussed method. The heat transfer equation was solved by the implicit finite-difference method of second-order accuracy in space. The time-space location of isotherms $X_i(t)$ was computed numerically for a value of heat flux $q(t)$. A random error was inserted into data obtained for the experiment error simulation. On the next step the heat flux evaluation problem was solved with using $X_i(t)$, obtained on the previous stage. The computed heat flux was compared with the initial one ($q(t)$).

The computational experiments results are provided in Figs. 3 and 4. The results of heat flux density restoration by using one and 10 isotherms in comparison with the exact solution are depicted in Fig. 3. As the initial guess, a constant heat flux was used. The influence of random error with value about 1% is presented too. In Fig. 4 the results of computational experiments for the heat flux having two maxima are depicted. The influence of random error with the value about 5% in the initial data ($X_i(t)$) is shown as well. The numerical experiments confirmed the opportunity of heat flux evaluation by using the measured isotherms. The increment of registered isotherms number improves the result, quite naturally, and increases the computation time. For the problem class considered here, 10 temperature indicators are quite

enough for the admissible accuracy. The influence of initial data error on the results' accuracy (the solution sensitivity to data errors) is considered to be acceptable. The apparent solution instability was not found in the computational experiments, despite our problem being ill-posed. It seems that the gradient method's regularizing properties [3] were sufficient for regularization of our problem.

DISCUSSION

Under the standard method of using set temperature indicators, the indicators being located in the plane of isotherms (on the surface for example), we can record the given indicator color being changed or not as a function of time. It is a piece-wise constant function and the intervals of constant color do not contain useful information. In comparison with this way, the method offered in given work has some significant advantages. Every indicator strip provides more information about the process: continuous functions $X_i(t)$ contain information on the heating at every moment. The chance to achieve threshold of sensitivity is far less, because in the sensor depth the temperature variation range is smaller. The temperature determination error is governed in this event by the color change error ΔT (about 1% [1]), but not the interval between adjacent indicators (it provides error about 10%). The sensor under the consideration has rather apparent qualitative distinctions from existing ones. The responsivity of the isotherms to the heat flux depends upon the depth and rapidly decreases when the distance to the surface increases. Thus, the sensor will have varying sensitivity depending on time. The signal recorded by this sensor has an integral character and thus it is stable. For this sensor, records decoding the system of data processing are necessary,

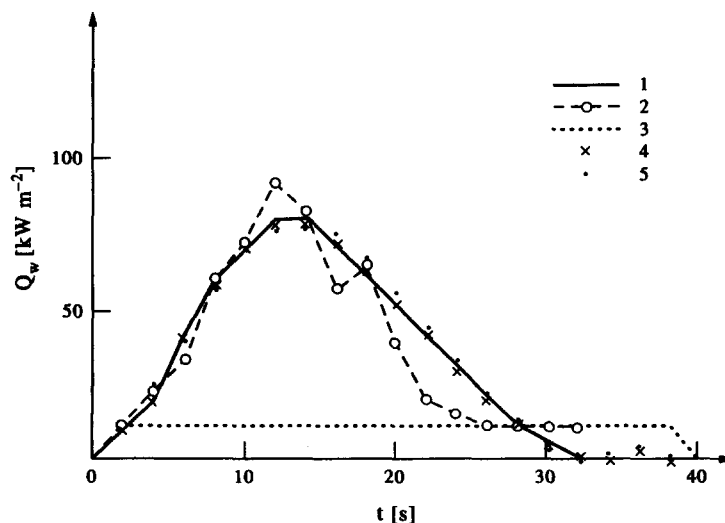


Fig. 3. The heat flux restoration results with one and 10 isotherms used. 1—exact solution; 2—result of computation using one isotherm; 3—initial guess; 4—result of computation using 10 isotherms; 5—result of computation using 10 isotherms (the error value is 1%).

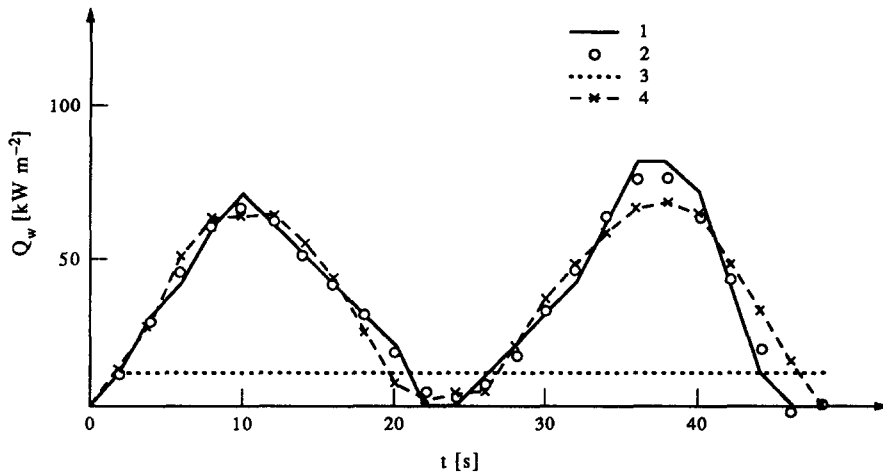


Fig. 4. The heat flux restoration results with 10 isotherms used. 1—exact solution; 2—result of computation; 3—initial guess; 4—result of computation using 10 isotherms (the error value is 5%).

which should include the inverse heat conduction problem (IHCP) solution, because direct interpretation of these data is impossible (excluding the quasi steady-state event).

CONCLUSIONS

The heat flux measurement method, in some sense alternative to existing ones, is considered. The method uses the data on isotherms for the boundary condition recovery. The method does not use the electric measurements. The heat flux record is not local in time and, therefore, the signal processing demands the IHCP solution. The number of isotherms we need is limited and not too big (far less than the number necessary for precise temperature field $T(t, x)$ restoration).

The application of reversible temperature indicators restricts the sensor utilization range as to maximal temperature, but it seems possible to upgrade it, replacing the temperature indicators with fusible inserts in order to eliminate this restriction. The problem analysis and the numerical experiments confirm the opportunity of optical heat flux sensor creation, based on the isotherms records in the coordinate-time plane.

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